Dimensionality of observable space

Sergei Viznyuk

I show the dimensionality of observable space is conditioned on objectivity. I explain distinction between measuring device and observer

Commonly, the implications of *objectivity*[1] are either overlooked or misappropriated. E.g., the Schrödinger equation is postulated [1, 2] like primordial law, instead of being derived from objectivity-imposed unitarity [3]. Here I show the objectivity also effectuates the dimensionality of observable space.

The question of why the observable space is three-dimensional, and if there are extra dimensions, is listed as one of the major unresolved problems of physics [4]. The currently existing propositions largely fall under three categories:

1. As exemplified by quote from [5]:
   Quantum (and classical) binding energy considerations in n-dimensional space indicate that atoms (and planets) can only exist in three-dimensional space. This is why observable space is solely 3-dimensional
2. Extra dimensions are compactified microscopically in a form of so-called "Calabi-Yau Spaces" [6, 7], stipulated by assortment of string theories [8, 9]
3. Immediately after the moment of creation, popularly named Big Bang, the things went bad for all but $D = 3$ dimensions [10, 11, 12]

A brief excursion into each category suffices to illustrate why the question is still open:

1. There is ample argument [13, 14, 15, 16, 17], that familiar $3D$ objects can't exist in $D \neq 3$ space. That would include virtually anything, from atoms to planets to any known form of matter. Yet, it does not prove in any way that other forms of matter and objects, including intelligent life, may not exist, governed by vastly different laws of [classical] physics
2. *No one has managed to extract any sort of experimental prediction out of the [string] theory other than that the cosmological constant should probably be at least 55 orders of magnitude larger than experimental bounds* [18]. In string theories, we deal with complete absence of factual basis. Below I point to the core reason for the failure of string theories to comply with empirical evidence. As it stands, any proposition based on a string theory is an unsubstantiated speculation
3. Even more speculative are propositions [10, 11] referring to the immediate aftermath of the moment of creation, Big Bang. No such hypothesis has any chance of experimental confirmation. Any theory not rendering itself to empirical validation falls out of scientific methodology into the realm of religion

And yet, the answer to a basic question must not be convoluted and impossible to attest to[2]. We just have to look at fundamentals of measurement, missed in numerous publications on the subject.

---

[1] The objectivity is defined as independence of objective facts on observer [basis]; *objective facts* being synonymous to classical information

[2] Transformation of question into answer effectively is a measurement on input state (question). A question, expressed in terms of M fundamental notions, has an answer expressed in terms of no more than M fundamental notions

It appears to be a common practice to use the term *space[time] dimensionality* [16, 15, 17] with no definition. Authors think the notion is so obvious that providing definition is superfluous. However, operating with undefined notion is even less credible endeavor than theorizing off a false assumption. Therefore, I start with definition. The space dimensionality is the cardinality of complete[3] observation operator basis, which is the max number of mutually orthogonal information-extracting[4] configurations of measuring device[5]. The device configurations $\boldsymbol{A}$ and $\boldsymbol{B}$ are orthogonal if output $\boldsymbol{A}$ does not convey any information about output $\boldsymbol{B}$. Formally, the orthogonality condition is:

$$Tr(\boldsymbol{AB}) = 0 \tag{1}$$

, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are traceless Hermitian operators. The device configuration has to be represented by a traceless operator, because traced part of a Hermitian operator, up to an invariant factor, is identity operator $\boldsymbol{I}$. Since output of $\boldsymbol{I}$ is same for any input, $\boldsymbol{I}$ does not extract[6] any information. Operator $\boldsymbol{I}$ outcome is the fact[7] of the measurement[8]. Only the traceless part of Hermitian operator extracts information [about input].

The space dimensionality, defined above, is related to cardinality M of measurement outcomes in *defining representation*[9] [19] of measurement operator. There are M^2 real parameters defining measurement operator. The traceless condition reduces the number of parameters by 1, i.e., the number of real parameters specifying configuration of measuring device is $M^2 - 1$. If space dimensionality is D, then, by definition, there are D traceless Hermitian operators $\{\boldsymbol{X}_i\}$; each being orthogonal to other $D - 1$ operators:

$$Tr\left(\boldsymbol{X}_i \boldsymbol{X}_{j \neq i}\right) = 0 \quad ; \quad 1 \leq i, j \leq D \tag{2}$$

For a given $\boldsymbol{X}_i$ the above represents $D - 1$ real linear equations[10]. The non-trivial solution of (2), for $M^2 - 1$ real parameters defining $\boldsymbol{X}_i$, only exists if

$$D < M^2 \tag{3}$$

From (3), the max value of D is $M^2 - 1$. It corresponds to the complete measurement operator basis, commonly represented by generalized Gell-Mann operators $\{\boldsymbol{\lambda}_i\}$ [20, 21]:

$$\boldsymbol{X} = \sum_{i=1}^{M^2-1} \alpha_i \boldsymbol{\lambda}_i = (\boldsymbol{r}, \boldsymbol{\sigma}) \tag{4}$$

, where

---

[3] The completeness of basis is manifested through group-forming commutator relations [19] between basis operators
[4] With measurement defined as extraction of classical information [3]
[5] For now, I consider *measurement* synonymous to *observation*, to avoid introducing any awkward terminology, such as, e.g., "observation device". Few paragraphs down I shall draw a distinction between measurement and observation
[6] Operator $\boldsymbol{I}$ is perfectly able to distinguish orthogonal inputs $\boldsymbol{x}, \boldsymbol{y}$, since $\langle \boldsymbol{x}|\boldsymbol{I}|\boldsymbol{y}\rangle = 0$. It's just not able to encode its output as classical information, because output of $\boldsymbol{I}$ is the same for any input: $\langle \boldsymbol{x}|\boldsymbol{I}|\boldsymbol{x}\rangle = \langle \boldsymbol{y}|\boldsymbol{I}|\boldsymbol{y}\rangle = 1$. The information-producing measurement can only be performed by operator whose output is different for different inputs
[7] A competed measurement event, e.g., a click of a particle detector, implicitly includes $\boldsymbol{I}$ [29]
[8] Sometimes it is incorrectly stated [19] that $\boldsymbol{I}$ is "do nothing" operator
[9] Also called the *fundamental* representation [30]
[10] Trace of a product of two Hermitian operators is always real

$$Tr(\boldsymbol{\lambda}_i\boldsymbol{\lambda}_j) = 2\delta_{ij} \;\; ; \;\; \alpha_i = Tr(\boldsymbol{X}\boldsymbol{\lambda}_i)/2 \tag{5}$$

$$\boldsymbol{r} \equiv 2 \cdot (\alpha_1, \dots, \alpha_{M^2-1}) \;\; ; \;\; \boldsymbol{\sigma} \equiv (\boldsymbol{\lambda}_1, \dots, \boldsymbol{\lambda}_{M^2-1})/2 \tag{6}$$

Thus, the space dimensionality is:

$$D = M^2 - 1 \tag{7}$$

It immediately follows, D can only take values $D = 0,3,8,15$ ... [22]. Other numbers of spatial dimensions, as proposed by now largely defunct theories, such as Kaluza-Klein [23] ($D = 4$) or *M*-theory [6, 18, 12] ($D = 10$) do not correspond to a complete measurement basis, and, therefore, do not make a *space*[11]. E.g., there is no $2D$ space, since two information-extracting operators do not make a complete basis. Any $2D$ or $1D$ model implies projection from $D = 3,8,15, \dots$.

Everyday experience points to the fact that we live in $D = 3$ space, corresponding to $M = 2$, i.e., qubit measurement basis. The unambiguous relations: $false = -true$; $left = -right$; $top = -bottom$; $not\ exist = -exist$, where minus sign means *objectively opposite*, are only possible in $M = 2$ basis. Had we lived in $M = 3$ world, there would be $D = 8$ spatial dimensions, mandating different laws of classical physics [16]. None of the known states of matter or objects would exist [17]. Furthermore, in $M > 2$ world nothing could be objectively ascertained, as one cannot unambiguously assign $true/false$ to $M > 2$ mutually exclusive measurement outcomes, similar to how we routinely assign $true = 1$ and $false = 0$ in $M = 2$ basis. Unlike $SO(3) \to SU(2)$ map, the $SO(D)$ transformations, which include spatial rotations, do not homomorphically map[12] onto $SU(M)$, for $M > 2$. It means, the extracted by measurement information, expected to be objective, for observer living in $D > 3$ space, depends on orientation of observer basis. Consequently, there is no objectivity in $M > 2$ world, and therefore, could not be an observer[13].

Having inferred that observer, as a notion, can only exist in $D = 3$ space, I thus arrived at *anthropic principle* [24, 25, 13]. Yet it does not mean the measurement basis is limited to $M = 2$. There is no reason[14] to pick any particular M over $M' > M$, given $SU(M)$ transformation group is a subgroup of a bigger $SU(M')$. Measurements in all $M = 1,2,3,4, \dots$ bases are legitimate and, therefore, omnipresent. What happens to dimensions other than those corresponding to $M = 2$ basis? For $M = 1$ the answer is obvious: measurement in $M = 1$ basis results in the same outcome for any input, and, therefore, produces no information to observe, i.e., no dimension, $D = 0$. For $M > 2$ bases I could resort to a primitive interpretation of anthropic principle, by stating that the known world is not feasible in $D \neq 3$, and thus claim "explanation" why we only see $D = 3$ dimensions. This is how many authors [17, 5, 15, 13, 16, 14] approach the subject[15]. However, the anthropic principle is not an exercise in tautology or circular reasoning. At the base of anthropic principle is the presence of observer, which implies objectivity. The objectivity signifies

---

[11] The space is defined as real vector space of measurement basis operators, i.e., the space of vectors $\boldsymbol{r}$ in (4)

[12] $SO(D)$ is defined by $(D^2 - D)/2 = (M^4 - 3M^2 + 2)/2$ real parameters, while $SU(M)$ is by $M^2 - 1$. They are equal only for $M = 2$

[13] The notion of observer is meaningless vis-a-vi observed world if outcome of observation is not objective

[14] There is no physical basis for *standard model* stopping at $SU(3)$, apart from the math becoming intractable and results uninterpretable, for higher M

[15] From [14]: *In the absence of a truly convincing argument* [about space dimensionality] *however we may rely instead on anthropic reasoning*

invariance of extracted by measurement classical information, formally enforced through unitarity [3, 26]. This is where comes the difference between measurement and observation. The measurement transforms quantum information into classical. Quantum and classical information are not invariants of measurement, only their sum is [26]. Contrary to some statements[16], the observer is not the measuring device. The observer only deals with extracted by measurement classical information, in a form of various classical objects, which make up the objective reality. The *representation* of classical information in observer basis (O-basis) is the *observation*. Unlike measurement, the observation conserves both quantum and classical information[17]. The information extracted by measurement in any M-basis, is represented, i.e., observed, in O-basis. Therefore, the dimensionality of observable space is the cardinality of observation operator basis, not of measurement operator basis. I prove below, the common O-basis for information extracted by measurements in all $M > 1$ bases can only be of cardinality $O = 2$, and corresponding space dimensionality (7) can only be $D = O^2 - 1 = 3$.

All basis operators $\{\boldsymbol{\lambda}_i\}$ in (4) only have 2 distinct non-zero eigenvalues for any $M > 1$ basis. Among $\{\boldsymbol{\lambda}_i\}$ are $M - 1$ commuting, i.e., simultaneously diagonalizable. These diagonal operators are encoding operators. They unambiguously assign input eigenstate a classical outcome - an eigenvalue (encoding symbol), whenever there is an output, i.e., non-zero eigenvalue. A measurement, whose output (a classical information) is encoded as one of 2 distinct symbols is effectively the measurement on [generalized] qubit[18]. The outcome of a measurement in M-basis is thus encoded as tensor product of outcomes of measurement on $M - 1$ qubits. E.g., the possible outcomes $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ of a measurement in $M = 4$ basis are encoded[19] as:

$$\boldsymbol{a} = \boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{A} \quad ; \quad \boldsymbol{b} = \boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{B} \quad ; \quad \boldsymbol{c} = \boldsymbol{A} \otimes \boldsymbol{B} \otimes \boldsymbol{B} \quad ; \quad \boldsymbol{d} = \boldsymbol{B} \otimes \boldsymbol{B} \otimes \boldsymbol{B} \tag{8}$$

, where $\{\boldsymbol{A}, \boldsymbol{B}\}$ are qubit measurement outcomes. Effectively, the cardinality M of measurement outcomes is the number of ways to distribute N identical inputs into O distinct observation bins. Therefore, the common O-basis for measurements in all $M > 1$ bases, has to satisfy:

$$for\ a\ given\ O\ and\ \forall(M > 1)\ \exists(N > 0) :\ M = \frac{(N + O - 1)!}{N!\,(O - 1)!} \tag{9}$$

The above holds only if $O = 2$. Q.E.D.

I have proved $D = 3$ is the only dimensionality of observable space wherein the existence of observer, and observation of outcomes of a measurement in all cardinality M bases is possible. The $3D$ dimensionality is effectuated by [objective] representation of classical information extracted by measurements in all cardinality M bases. I have shown the outcome of a measurement in any M-basis is a tensor product of outcomes of a measurement on $M - 1$ generalized qubits. I have explained the distinction between measurement and observation. The objectivity-imposed

[16] From [13]: *Even quantum mechanics, which supposedly brought the observer into physics, makes no use of intellectual properties; a photographic plate would serve equally well as an 'observer'*
[17] That answers the question, if, by observing Moon, we make its wave function collapse; or if there was a sound of a tree falling, if no one listened
[18] In $M = 3$ case, the definition of generalized qubit measurement operator effectively merges definitions of isospin and hypercharge operators, since both have 2 distinct non-zero eigenvalues
[19] The order of qubits in tensor products (8) is irrelevant

unitarity [3] may only pertain to transformation of observation basis, not necessarily to transformation of measurement basis (configuration of measuring device). In accordance with everyday experience, the observation basis is expected to transform under M-dimensional representation of $SU(2)$ group [27], [locally] isomorphic to $SO(3)$ [28]. The configuration of measuring device is not under such restriction, and, generally, is expected to only abide the intertwist relation [26]:

$$\boldsymbol{T}\boldsymbol{X}' = \boldsymbol{X}\boldsymbol{T} \tag{13}$$

, where $\boldsymbol{X}$ and $\boldsymbol{X}'$ are device configurations measuring the same input, and $\boldsymbol{T}$ is transformation from $\boldsymbol{X}$ to $\boldsymbol{X}'$. The unitarity of $\boldsymbol{T}$ follows from (13) only if $M = 2$ and $det(\boldsymbol{T}) \neq 0$ [26].

References

[1] M. Nielsen and I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2010.

[2] M. Kober, "Copenhagen Interpretation of Quantum Theory and the Measurement Problem," *arXiv:0905.0408 [physics.hist-ph]*, 2009.

[3] S. Viznyuk, "The origin of unitary dynamics," 2020. [Online]. Available: https://www.academia.edu/44809958/The_origin_of_unitary_dynamics.

[4] "List of unresolved problems in physics," Wikipedia, [Online]. Available: https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics. [Accessed 12 2021].

[5] M. Rabinowitz, "Why Observable Space Is Solely Three Dimensional," *arXiv:1502.04989 [physics.gen-ph]*, 2014.

[6] M. Duff, "M-Theory (the Theory Formerly Known as Strings)," *arXiv:hep-th/9608117*, 1996.

[7] B. Greene, "String Theory on Calabi-Yau Manifolds," *arXiv:hep-th/9702155*, 1997.

[8] R. Hedrich, "String Theory - From Physics to Metaphysics," *arXiv:physics/0604171 [physics.hist-ph]*, 2006.

[9] T. Weigand, "Introduction to String Theory," *https://www.thphys.uni-heidelberg.de/courses/weigand/Strings11-12.pdf*, 2011.

[10] J. Gonzalez-Ayala and F. Angulo-Brown, "Is the (3 + 1) − d nature of the universe a thermodynamic necessity," *arXiv:1502.01843 [gr-qc]*, 2015.

[11] S. Kim, J. Nishimura and A. Tsuchiya, "Late time behaviors of the expanding universe in the IIB matrix model," *arXiv:1208.0711 [hep-th]*, 2012.

[12] F. Klinkhamer, "M-theory and the birth of the Universe," *arXiv:2102.11202 [hep-th]*, 2021.

[13] J. Barrow and F. Tipler, The anthropic cosmological principle, Oxford: Oxford University Press, 1986.

[14] J. Scargill, "Can Life Exist in 2 + 1 Dimensions?," *arXiv:1906.05336 [hep-th]*, 2019.

[15] J. Barrow, "Dimensionality," *Phil. Trans. R. Soc. Lond.,* vol. A 310, pp. 337-346, 1983.

[16] M. Tegmark, "On the dimensionality of spacetime," *arXiv:gr-qc/9702052,* 1997.

[17] C. Petway, R. Orlando, A. McNamara, E. Zweig, B. Caminada, E. Kincaid, C. Landgraf, C. Mohs, M. Schiff and E. Fischbach, "Constraints on the Dimensionality of Space," *arXiv:2105.05311 [physics.class-ph]*, 2021.

[18] P. Woit, "String Theory: An Evaluation," *arXiv:physics/0102051 [physics.soc-ph]*, 2001.

[19] H. Georgi, Lie Algebras in Particle Physics. From Isospin to Unified Theories, Boca Raton: CRC Press, Taylor & Francis Group, 2018.

[20] R. Bertlmann and P. Krammer, "Bloch vectors for qudits," *arXiv:0806.1174 [quant-ph],* 2008.

[21] J. Patera and H. Zassenhaus, "The Pauli matrices in n dimensions and finest gradings of simple Lie algebras of type An-1," *J. Math. Phys.,* vol. 29, pp. 665-673, 1988.

[22] N. Sloane, "OEIS sequence A005563," 2010. [Online]. Available: http://oeis.org/A005563.

[23] M. Tajmar and L. Williams, "An Experimental Test of the Classical Interpretation of the Kaluza Fifth Dimension," *Physics,* vol. 2, no. 4, p. 587–595, 2020.

[24] B. Carter, "Anthropic Principle in Cosmology," *arXiv:gr-qc/0606117,* 2006.

[25] B. Carter, "Anthropic interpretation of quantum theory," *arXiv:hep-th/0403008,* 2004.

[26] S. Viznyuk, "Where unfathomable begins," Academia.edu, 2021. [Online]. Available: https://www.academia.edu/45647801/Where_unfathomable_begins.

[27] N. Wheeler, "Spin matrices for arbitrary spin," *https://www.reed.edu/physics/faculty/wheeler/documents/Quantum%20Mechanics/Miscellaneous%20Essays/Angular%20Momentum,%20Spin/D3.%20Spin%20Matrices.pdf,* 2000.

[28] J. Atkins, "SU(2)’s Double-Covering of SO(3)," 12 12 2019. [Online]. Available: https://www.pas.rochester.edu/assets/pdf/undergraduate/su-2s_double_covering_of_so-3.pdf. [Accessed 11 2021].

[29] S. Viznyuk, "No decoherence by entanglement," 2020. [Online]. Available: https://www.academia.edu/43260697/No_decoherence_by_entanglement.

[30] M. Steinkirch, Introduction to Group Theory for Physicists, New York, 2011.