The basics of information transmission

Sergei Viznyuk

There is a glaring contradiction, duly noted in [1], between ubiquity of transmission of classical information in everyday life, and multitude of no-go theorems, such as no-cloning [2, 3, 4], no-teleportation [5], no-broadcasting [6], no-communication [7], no-signaling [8], which disallow information, contained in arbitrary state, from being copied. The unfeasibility could be ascertained multiple ways:

- Suppose there is a linear operator $\boldsymbol{U}$ able to clone an arbitrary normalized state $\boldsymbol{A}$ over target $\boldsymbol{S}$: $\boldsymbol{U}|\boldsymbol{AS}\rangle = |\boldsymbol{AA}\rangle$. Since state $\boldsymbol{A}$ is arbitrary, operator $\boldsymbol{U}$ is also able to clone orthogonal to $\boldsymbol{A}$ normalized state $\boldsymbol{B}$: $\boldsymbol{U}|\boldsymbol{BS}\rangle = |\boldsymbol{BB}\rangle$, as well as normalized state $\boldsymbol{\psi} = \alpha\boldsymbol{A} + \beta\boldsymbol{B}$: $\boldsymbol{U}|\boldsymbol{\psi S}\rangle$=$|\boldsymbol{\psi\psi}\rangle$=$\alpha^2|\boldsymbol{AA}\rangle$+$\beta^2|\boldsymbol{BB}\rangle + \alpha\beta|\boldsymbol{AB}\rangle + \beta\alpha|\boldsymbol{BA}\rangle$. From linearity of $\boldsymbol{U}$: $\boldsymbol{U}|\boldsymbol{\psi S}\rangle = \alpha|\boldsymbol{AA}\rangle$+$\beta|\boldsymbol{BB}\rangle \implies \alpha^2|\boldsymbol{AA}\rangle$+$\beta^2|\boldsymbol{BB}\rangle + \alpha\beta|\boldsymbol{AB}\rangle + \beta\alpha|\boldsymbol{BA}\rangle = \alpha|\boldsymbol{AA}\rangle$+$\beta|\boldsymbol{BB}\rangle$. Taking scalar product with $\langle\boldsymbol{AA}|$ of both sides of last equation, I obtain $\alpha^2 = \alpha$ which is only possible if $\alpha = 1$ or 0. Thus, operator $\boldsymbol{U}$ is not able to clone an arbitrary state. In particular, it is not able to copy state $\boldsymbol{\psi} = \alpha\boldsymbol{A} + \beta\boldsymbol{B}$, where $\alpha, \beta \neq 0{,}1$. Also, contrary to statement "*a cloning device can only clone states which are orthogonal*" on page 532 [1], it is easy to see that it is not able to clone orthogonal to $\boldsymbol{\psi} = \alpha\boldsymbol{A} + \beta\boldsymbol{B}$ state $\boldsymbol{\phi} = \alpha|\beta/\alpha|\boldsymbol{A} - \beta|\alpha/\beta|\boldsymbol{B}$
- Cloning of arbitrary state would enable measurement, i.e., extraction of classical information, to be performed on replicas without destroying the original state. Such proposition is impossible in quantum theory

Could it be that cloning is possible for particular states, such as eigenstates of measuring device? It is widely assumed (see, e.g., exercise 12.1 in [1]), that such states can be easily cloned via quantum channel. As I show below, that is also not the case.

- If $\boldsymbol{A}$ and $\boldsymbol{B}$ are eigenstates of measuring device, they are unitarity equivalent to superposed normalized states $\boldsymbol{\psi} = \alpha\boldsymbol{A} + \beta\boldsymbol{B}$, and $\boldsymbol{\phi} = \alpha|\beta/\alpha|\boldsymbol{A} - \beta|\alpha/\beta|\boldsymbol{B}$. Namely, there exist unitary transformation $\boldsymbol{V}$, such that $\boldsymbol{A} = \boldsymbol{V\psi}$ and $\boldsymbol{B} = \boldsymbol{V\phi}$. If we assume there exists linear operator $\boldsymbol{U}$ able to copy $\boldsymbol{A}$ over target $\boldsymbol{S}$: $\boldsymbol{U}|\boldsymbol{AS}\rangle = |\boldsymbol{AA}\rangle$, then $\boldsymbol{U}|\boldsymbol{AS}\rangle = \boldsymbol{U}|\boldsymbol{V\psi S}\rangle = |\boldsymbol{AA}\rangle = |\boldsymbol{V\psi}, \boldsymbol{V\psi}\rangle \implies \boldsymbol{UV}|\boldsymbol{\psi S}\rangle = \boldsymbol{VV}|\boldsymbol{\psi\psi}\rangle$. Thus, based on our assumption, there exists linear operator $\boldsymbol{U'} = \boldsymbol{V}^\dagger\boldsymbol{V}^\dagger\,\boldsymbol{UV}$ able to copy state $\boldsymbol{\psi} = \alpha\boldsymbol{A} + \beta\boldsymbol{B}$ over target $\boldsymbol{S}$: $\boldsymbol{U'}|\boldsymbol{\psi S}\rangle$=$|\boldsymbol{\psi\psi}\rangle$, which has been proven impossible above.

This result may seem controversial, as eigenstates of measuring device are classical, in a sense, they are not destroyed by measurement, and therefore, can be cloned. However, no-go theorems and ascertainment in previous paragraph only apply to cloning via quantum channel, i.e., via unitary operation, not via classical channel. It has been shown [9], no information is transmitted via quantum channel. Hence, no cloning is possible via quantum channel only. Teleportation or cloning always involves transmission of information via classical channel. Then, what exactly is *classical channel*? That is the question I answer in this paper.

Consider a generic setup with Alice sending a message to Bob. The available operations are unitary transformation, which preserves both quantum and classical information, and *measurement*, which converts quantum information into classical. There must be a shared state to perform these operations on. Consider a basic shared state of two entangled qubits, one accessible to Alice and another accessible to Bob:

$$\boldsymbol{\Psi} = (|\boldsymbol{\psi u}\rangle + |\boldsymbol{\phi v}\rangle)/\sqrt{2} \;\; ; \qquad \boldsymbol{\rho} = |\boldsymbol{\Psi}\rangle\langle\boldsymbol{\Psi}| \tag{1}$$

Here $\boldsymbol{\psi}, \boldsymbol{\phi}$ are Alice's entangled qubit states, and $\boldsymbol{u}, \boldsymbol{v}$ are Bob's qubit states. For $|\boldsymbol{\psi u}\rangle$ and $|\boldsymbol{\phi v}\rangle$ in (1) to be orthogonal, I consider $\boldsymbol{\psi}, \boldsymbol{\phi}$ normalized, but not necessarily orthogonal, and $\boldsymbol{u}, \boldsymbol{v}$ normalized and orthogonal:

$$\langle\boldsymbol{\psi}|\boldsymbol{\psi}\rangle = 1 \; ; \;\; \langle\boldsymbol{\phi}|\boldsymbol{\phi}\rangle = 1 \; ; \; \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle \neq 0 \; ; \; \langle\boldsymbol{u}|\boldsymbol{u}\rangle = 1 \; ; \;\; \langle\boldsymbol{v}|\boldsymbol{v}\rangle = 1 \; ; \; \langle\boldsymbol{u}|\boldsymbol{v}\rangle = 0$$

The amount of information extracted from shared state with measurement performed by Alice is given by Von Newmann's entropy $H_A = -Tr(\boldsymbol{\rho}_A \log_2 \boldsymbol{\rho}_A) \;\; bits/event$, where $\boldsymbol{\rho}_A$ is shared state with Alice's part traced out:

$$\boldsymbol{\rho}_A = Tr_A(\boldsymbol{\rho}) = (|\boldsymbol{u}\rangle\langle\boldsymbol{u}| + \langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|\boldsymbol{u}\rangle\langle\boldsymbol{v}| + \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle|\boldsymbol{v}\rangle\langle\boldsymbol{u}| + |\boldsymbol{v}\rangle\langle\boldsymbol{v}|)/2 \tag{2}$$

$$H_A = -Tr(\boldsymbol{\rho}_A \log_2 \boldsymbol{\rho}_A) =$$

$$1 - \frac{(1 + |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|)}{2} \log_2(1 + |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|) - \frac{(1 - |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|)}{2} \log_2(1 - |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|) \;\; bits/event$$

, with $|\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|^2 = 1 - 4 \cdot det(\boldsymbol{\rho}_A)$, $|\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|$ being the length of Bloch vector [10, 11]. Product $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$ tells how reliably can Alice's device distinguish $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ states. If $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle = 0$, then $\boldsymbol{\psi}$, $\boldsymbol{\phi}$ are perfectly distinguishable, i.e., orthogonal, in Alice's measurement basis. In this case, $H_A = 1 \; bits/event$. If $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle \neq 0$, Alice's device cannot perfectly distinguish $\boldsymbol{\psi}$, $\boldsymbol{\phi}$ states, as in case, e.g., when $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ are horizontal and vertical polarizations of a photon, with non-ideal *PBS* (polarizing beam splitter) used to separate them (Figure 1 in [10]). In this case $H_A < 1 \; bits/event$. In particular, if $|\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle| = 1$, then $H_A = 0$. Thus, the amount of information extracted by Alice depends on orthogonality of entangled Alice's states.

If Bob's measurement basis is $\boldsymbol{u}, \boldsymbol{v}$, then expectation value $\langle\boldsymbol{X}_B\rangle$ of Bob's measurement operator $\boldsymbol{X}_B = X_u|\boldsymbol{u}\rangle\langle\boldsymbol{u}| + X_v|\boldsymbol{v}\rangle\langle\boldsymbol{v}|$ is: $\langle\boldsymbol{X}_B\rangle = Tr(\boldsymbol{X}_B\boldsymbol{\rho}) = Tr(\boldsymbol{X}_B\boldsymbol{\rho}_A) = (X_u + X_v)/2$. It does not depend on Alice's measurement, as stipulated by no-communication and no-signaling theorems [7, 8]. However, if Bob chooses to do his measurement in $\mathbf{0}, \mathbf{1}$ basis, wherein $|\boldsymbol{u}\rangle = |\mathbf{0}\rangle\langle\mathbf{0}|\boldsymbol{u}\rangle + |\mathbf{1}\rangle\langle\mathbf{1}|\boldsymbol{u}\rangle$ and $|\boldsymbol{v}\rangle = |\mathbf{0}\rangle\langle\mathbf{0}|\boldsymbol{v}\rangle + |\mathbf{1}\rangle\langle\mathbf{1}|\boldsymbol{v}\rangle$, then from (2) we would have:

$$\begin{aligned}
\boldsymbol{\rho}_A = {} & \frac{1}{2}(|\langle\mathbf{0}|\boldsymbol{u}\rangle|^2 + |\langle\mathbf{0}|\boldsymbol{v}\rangle|^2 + \langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle\langle\boldsymbol{v}|\mathbf{0}\rangle\langle\mathbf{0}|\boldsymbol{u}\rangle + \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle\langle\boldsymbol{u}|\mathbf{0}\rangle\langle\mathbf{0}|\boldsymbol{v}\rangle)|\mathbf{0}\rangle\langle\mathbf{0}| + \\
& \frac{1}{2}(\langle\mathbf{0}|\boldsymbol{u}\rangle\langle\boldsymbol{u}|\mathbf{1}\rangle + \langle\mathbf{0}|\boldsymbol{v}\rangle\langle\boldsymbol{v}|\mathbf{1}\rangle + \langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle\langle\mathbf{0}|\boldsymbol{u}\rangle\langle\boldsymbol{v}|\mathbf{1}\rangle + \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle\langle\mathbf{0}|\boldsymbol{v}\rangle\langle\boldsymbol{u}|\mathbf{1}\rangle)|\mathbf{0}\rangle\langle\mathbf{1}| + \\
& \frac{1}{2}(\langle\mathbf{1}|\boldsymbol{u}\rangle\langle\boldsymbol{u}|\mathbf{0}\rangle + \langle\mathbf{1}|\boldsymbol{v}\rangle\langle\boldsymbol{v}|\mathbf{0}\rangle + \langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle\langle\mathbf{1}|\boldsymbol{u}\rangle\langle\boldsymbol{v}|\mathbf{0}\rangle + \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle\langle\mathbf{1}|\boldsymbol{v}\rangle\langle\boldsymbol{u}|\mathbf{0}\rangle)|\mathbf{1}\rangle\langle\mathbf{0}| + \\
& \frac{1}{2}(|\langle\mathbf{1}|\boldsymbol{u}\rangle|^2 + |\langle\mathbf{1}|\boldsymbol{v}\rangle|^2 + \langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle\langle\boldsymbol{v}|\mathbf{1}\rangle\langle\mathbf{1}|\boldsymbol{u}\rangle + \langle\boldsymbol{\psi}|\boldsymbol{\phi}\rangle\langle\boldsymbol{u}|\mathbf{1}\rangle\langle\mathbf{1}|\boldsymbol{v}\rangle)|\mathbf{1}\rangle\langle\mathbf{1}|
\end{aligned} \tag{3}$$

To simplify (3), consider $|\boldsymbol{u}\rangle = (|\mathbf{0}\rangle + |\mathbf{1}\rangle)/\sqrt{2}$, and $|\boldsymbol{v}\rangle = (|\mathbf{0}\rangle - |\mathbf{1}\rangle)/\sqrt{2}$. With that:

$$\boldsymbol{\rho}_A = \frac{1}{2}(1 + Re\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle)|\mathbf{0}\rangle\langle\mathbf{0}| - \frac{i}{2}Im\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|\mathbf{0}\rangle\langle\mathbf{1}| + \frac{i}{2}Im\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|\mathbf{1}\rangle\langle\mathbf{0}| + \frac{1}{2}(1 - Re\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle)|\mathbf{1}\rangle\langle\mathbf{1}| \quad (4)$$

, where $Re\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$ and $Im\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$ are real and imaginary parts of $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$.
The expectation value of Bob's measurement operator $\boldsymbol{X}_B = X_0|\mathbf{0}\rangle\langle\mathbf{0}| + X_1|\mathbf{1}\rangle\langle\mathbf{1}|$ in $\mathbf{0}, \mathbf{1}$ basis is:

$$\langle\boldsymbol{X}_B\rangle = Tr(\boldsymbol{X}_B\boldsymbol{\rho}) = Tr(\boldsymbol{X}_B\boldsymbol{\rho}_A) = \frac{X_0 + X_1}{2} + \frac{X_0 - X_1}{2}Re\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle \quad (5)$$

As transpired, Alice can alter expectation value (5) of Bob's measurement by tuning (modulating) her own measuring device (transmitter), i.e., by controlling $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$ product. This mechanism, involving modulation of transmitter, and measurement of the reduced shared state by receiving device, constitutes classical communication channel. The actual results of any measurement are not transmitted. It underscores the absoluteness of classical information [10], in that information can only be extracted once, and cannot be re-extracted again from any state or "channel". To illustrate the point, consider a shared entangled state $\boldsymbol{\rho}$ of $\boldsymbol{A}$ and $\boldsymbol{B}$ [qubits, qdits], accessible respectively, to Alice and Bob. It's not too difficult to see that reduced states $\boldsymbol{\rho}_A = Tr_A(\boldsymbol{\rho})$ and $\boldsymbol{\rho}_B = Tr_B(\boldsymbol{\rho})$ are unitarily equivalent, i.e., $\exists\boldsymbol{U}$, such that $\boldsymbol{\rho}_B = \boldsymbol{U}\boldsymbol{\rho}_A\boldsymbol{U}^\dagger$; $\boldsymbol{U}\boldsymbol{U}^\dagger = \boldsymbol{I}$. In case of (1), it is obvious from diagonal form of $\boldsymbol{\rho}_A$ and $\boldsymbol{\rho}_B$:

$$diag(\boldsymbol{\rho}_A) = diag(\boldsymbol{\rho}_B) = \frac{1}{2}(1 + |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|)|\mathbf{0}\rangle\langle\mathbf{0}| + \frac{1}{2}(1 - |\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle|)|\mathbf{1}\rangle\langle\mathbf{1}|$$

The shared state [of entangled qubits] is a crucial element of any communication. Such state could be the output state of higher cardinality ($M > 2$) measurement operator, encoded in $M - 1$ entangled qubits [12]. Entangled qubits make up elements of objective reality, which transmitter and receiver are parts of. The transmitter must be in correlated (entangled) relationship with the receiver for communication to work, thuswise objective reality to be *observable* [13].

Alice's or Bob's measurement reduces shared state (1) to unitarily equivalent $\boldsymbol{\rho}_A$ or $\boldsymbol{\rho}_B$. Yet, if Alice's and Bob's devices are spacetime separated, their measurement operators are not unitarily equivalent. It is obvious from spacetime $(t, \boldsymbol{r}) = (t, x, y, z)$ parameterization of qubit operator: $\boldsymbol{X} = t \cdot \boldsymbol{I} + (\boldsymbol{r} \cdot \boldsymbol{\sigma})$. A change in t and/or $r = \sqrt{x^2 + y^2 + z^2}$ would result in a change to eigenvalues $[\epsilon_1 \ ; \ \epsilon_2] = [t + r \ ; \ t - r]$ of $\boldsymbol{X}$, and therefore, non-equivalent operator. The relationship between Hermitian operators $\boldsymbol{X}_A$, $\boldsymbol{X}_B$ having shared state, is *intertwist* $\boldsymbol{V}\boldsymbol{X}_A = \boldsymbol{X}_B\boldsymbol{V}$, where $\boldsymbol{V}$ is a linear transformation. For qubit operators, such transformation $\boldsymbol{V}$ only exists if [10]:

$$((t_A - t_B)^2 - \boldsymbol{r}_A^2 - \boldsymbol{r}_B^2)^2 = 4\boldsymbol{r}_A^2\boldsymbol{r}_B^2 \quad (6)$$

Three non-trivial situations satisfy (6):

1. $t_A = t_B \ ; \ \boldsymbol{r}_A^2 = \boldsymbol{r}_B^2$. In this case $\boldsymbol{V}$ is unitary; $\boldsymbol{X}_A$, $\boldsymbol{X}_B$ are unitarily equivalent, but not necessarily commuting
2. $t_A \neq t_B \ ; (t_A - t_B)^2 = (\boldsymbol{r}_A - \boldsymbol{r}_B)^2 = \boldsymbol{r}_A^2 + \boldsymbol{r}_B^2 - 2(\boldsymbol{r}_A, \boldsymbol{r}_B) \ ; |(\boldsymbol{r}_A, \boldsymbol{r}_B)| = r_A r_B$. In this case $\boldsymbol{V}$ is non-unitary; $\boldsymbol{X}_A$, $\boldsymbol{X}_B$ are commuting, but not unitarily equivalent

3. $t_A \neq t_B$; $t_A^2 = \boldsymbol{r}_A^2$; $t_B^2 = \boldsymbol{r}_B^2$. $\boldsymbol{V}$ is non-unitary; $\boldsymbol{X}_A$, $\boldsymbol{X}_B$ are not unitarily equivalent, and not necessarily commuting

The outcomes of measurement by Alice and Bob on shared state implicitly incorporate $\boldsymbol{c}^2 = 1$ speed limit in relationship between observables: $\boldsymbol{c}^2(t_A - t_B)^2 = (\boldsymbol{r}_A - \boldsymbol{r}_B)^2$ or $\boldsymbol{c}^2 t_A^2 = \boldsymbol{r}_A^2$; $\boldsymbol{c}^2 t_B^2 = \boldsymbol{r}_B^2$ whenever $t_A \neq t_B$. The speed limit is effectuated by Hermiticity of [qubit] measurement operators.

References

[1] M. Nielsen and I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2010.

[2] J. Parker, "The concept of transition in quantum mechanics," *Foundations of Physics,* vol. 1, no. 1, p. 23–33, 1970.

[3] W. Zurek and W. Wootters, "A single quantum cannot be cloned," *Nature,* vol. 299, no. 5886, p. 802–803, 1982.

[4] D. Dieks, "Communication by EPR devices," *Physics Letters A,* vol. 92, no. 6, p. 271–272, 1982.

[5] R. Werner, "Optimal Cloning of Pure States," *arXiv:quant-ph/9804001,* 04 1998.

[6] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa and B. Schumacher, "Noncommuting Mixed States Cannot Be Broadcast," *Physical Review Letters,* vol. 76, no. 15, p. 2818–2821, 1995.

[7] A. Peres and D. Terno, "Quantum Information and Relativity Theory," *arXiv:quant-ph/0212023,* 2003.

[8] T. De Angelis, F. De Martini, E. Nagali and F. Sciarrino, "Experimental test of the no signaling theorem," *arXiv:0705.1898 [quant-ph],* 2007.

[9] S. Viznyuk, "Kitchen experiment on entanglement and teleportation," 2020. [Online]. Available: https://www.academia.edu/42219989/Kitchen_experiment_on_entanglement_and_teleportation.

[10] S. Viznyuk, "Where unfathomable begins," Academia.edu, 2021. [Online]. Available: https://www.academia.edu/45647801/Where_unfathomable_begins.

[11] S. Haroche and J. Raymond, Exploring the Quantum, Oxford University Press, 2006.

[12] S. Viznyuk, "Ontogenesis, Part 1," 11 2023. [Online]. Available: https://www.academia.edu/109685658/Ontogenesis_Part_1.

[13] S. Viznyuk, "Dimensionality of observable space," Academia.edu, 2021. [Online]. Available: https://www.academia.edu/66663981/Dimensionality_of_observable_space.