Kitchen experiment on entanglement and teleportation

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Abstract

I prove teleportation protocol for an arbitrary qubit state can be implemented with one bit of information transmitted via classical channel, per preparation + measurement cycle. I show how teleportation protocol can be implemented in a classical setting. I discuss the contextual meaning of teleportation

Entanglement and teleportation have become popular buzzwords, generating interest in scientific community, as well as industry, and government, with promise of secure communication [1], and quantum computing capabilities [2].

Teleportation relies on a shared two-qubit entangled ancilla state between the sender (Alice), and the receiver (Bob). The subject of teleportation is the *third* qubit in unknown state, which Alice wants to teleport to Bob.

As entanglement is deemed a purely quantum feature, the teleportation is assumed to be only possible via quantum channel. The publicized teleportation schemes [3, 2] involve unitary transformations, a measurement by Alice in $4D$ basis on her two accessible qubits, and two bits of information transmitted by Alice to Bob via classical channel. All that effort is in order for Bob to reproduce the measurement result sample which would have been obtained by direct measurements of *third* qubit.

As an improvement over publicized teleportation schemes, I present teleportation protocol which only involves one projection transformation, one measurement by Alice, and $1\,bit$ of information transmitted by Alice to Bob via classical channel. I show that to the same end result, teleportation can be realized in a classical setting.

The teleportation protocol is based on Alice and Bob sharing entangled ancilla state ψ_{AB} of two qubits: $\psi_{AB} = \sum\rho_{ij}|i_A\rangle|j_B\rangle\,; i,j = \{0,1\}; \sum\rho_{ij}^{\dagger}\,\rho_{ij} = 1$. The subscripts A, B designate qubits accessible respectively to Alice and Bob. Alice and Bob are free to choose the shared ancilla state. A *third* qubit, in unknown state

$$\chi_A = \alpha|0_A\rangle + \beta|1_A\rangle \tag{1}$$

is given to Alice to teleport to Bob. In standard protocol, two unitary transformations ($\boldsymbol{CNOT}$-gate + $\boldsymbol{H}$-gate) [2], and two measurements are required for Alice to perform, and $2\,bits$ of information to be transmitted by Alice to Bob via classical channel per preparation + measurement cycle (*PMC*), in order for Bob to reconstruct the measurement result sample for the *third* qubit. Alice performs her measurements in cardinality $M = 4$ basis, the results of which require $\log_2 4 = 2\,bits$, to be passed to Bob, per PMC. There is, however, a redundancy in traditional protocol, as Bob's measurement is performed in cardinality $M = 2$ basis, so Alice's $4D$ Hilbert space, one way or another, gets projected into Bob's $2D$ space. In the proposed protocol this redundancy is eliminated.

To start, Alice and Bob choose the usual, maximally entangled shared ancilla state ψ_{AB}:

$$\psi_{AB} = \frac{|0_A 1_B\rangle + |1_A 0_B\rangle}{\sqrt{2}} \tag{2}$$

The proposed teleportation protocol is implemented in PMC steps as follows:

1. Preparation of the standard product state:

$$\begin{aligned}\Psi_{AB} &= \chi_A \otimes \psi_{AB} = (\alpha|0_A\rangle + \beta|1_A\rangle) \otimes \frac{|0_A 1_B\rangle + |1_A 0_B\rangle}{\sqrt{2}} \\ &= \frac{|0_A 0_A\rangle}{\sqrt{2}}\alpha|1_B\rangle + \frac{|1_A 1_A\rangle}{\sqrt{2}}\beta|0_B\rangle + \frac{|0_A 1_A\rangle}{\sqrt{2}}\alpha|0_B\rangle + \frac{|1_A 0_A\rangle}{\sqrt{2}}\beta|1_B\rangle\end{aligned} \tag{3}$$

Next, Alice wants to use observation basis of cardinality $M = 2$. She also wants the basis to be transformed, so the state of Bob's qubit would look separated and unitarily equivalent to the state of *third* qubit.

2. Alice transforms to cardinality $M = 2$ observation basis, with basis vectors:

$$\begin{aligned}|x_A\rangle &= \frac{|0_A 0_A\rangle + |1_A 1_A\rangle}{\sqrt{2}} \\ |y_A\rangle &= \frac{|0_A 1_A\rangle + |1_A 0_A\rangle}{\sqrt{2}}\end{aligned} \tag{4}$$

   Transformation of observation basis is performed with projection operator $\boldsymbol{F}_A$:

$$\begin{aligned}\boldsymbol{F}_A &= \sqrt{2}|x_A\rangle\langle y_A| + \sqrt{2}|y_A\rangle\langle x_A| \\ &= \frac{(|0_A 0_A\rangle + |1_A 1_A\rangle)(\langle 1_A 0_A| + \langle 0_A 1_A|)}{\sqrt{2}} \\ &+ \frac{(|0_A 1_A\rangle + |1_A 0_A\rangle)(\langle 0_A 0_A| + \langle 1_A 1_A|)}{\sqrt{2}}\end{aligned} \tag{5}$$

   Applying operator (5) to (3) results in:

$$\boldsymbol{F}_A \Psi_{AB} = \frac{|x_A\rangle(\alpha|0_B\rangle + \beta|1_B\rangle)}{\sqrt{2}} + \frac{|y_A\rangle(\alpha|1_B\rangle + \beta|0_B\rangle)}{\sqrt{2}} \tag{6}$$

   In (6), the Alice's qubits are in orthogonal states x_A and y_A, and Bob's qubit is in a separated superposition of state (1), and unitarily equivalent to (1) state $\alpha|1_B\rangle + \beta|0_B\rangle$.

3. Alice performs the measurement of accessible to her qubits of state (6). With equal probability Alice finds accessible to her qubits in state x_A, if both qubits are the same, or state y_A, if qubits are different.

4. If Alice finds her qubits to be identical, i.e. in x_A state, she sends Bob a single bit with value 0, meaning he has to measure his qubit without any transformation. If Alice finds her qubits in y_A state, she sends Bob a single bit with value 1, meaning Bob has to apply $\boldsymbol{X}$-gate on his qubit, swapping 0 and 1, before taking measurement. The $\boldsymbol{X}$-gate applied to $\alpha|1_B\rangle + \beta|0_B\rangle$ turns it into target state (1).

Repeating PMC steps 1-4 will let Bob obtain the same measurement result sample, as if he performed measurement on state (1), which, in minds of many authors, means the state (1) has been successfully teleported by Alice to Bob. The experimenters on quantum teleportation [4] reported success when measurement result samples matched with fidelity 0.8.

I shall now show the Bell-type entanglement and associated teleportation protocol can be implemented in purely classical setting, to achieve the same result as in quantum teleportation described above.

Bob invites Alice for a dinner and promises to entertain her with teleportation experiment. Bob prepares two pairs of identical matching gloves, and four black boxes, one per glove, and a mirror. Bob puts four gloves into four boxes, one glove per box, and closes boxes. Once Alice shows up, Bob gives boxes to Alice. He asks Alice to randomly shuffle boxes, behind his back, without looking into them, and then give one box to him. Thus, Bob gets one box, and Alice keeps three boxes. Bob asks Alice to put one of her three boxes aside. He then says, that he can tell which glove is in that box, if Alice opens her remaining two boxes and tells Bob only one thing: if the gloves she sees make a pair or not (i.e. if they are matching left and right gloves). If Alice tells Bob, she found left and right gloves in her remaining two boxes, Bob opens his box while looking into mirror image of its content (i.e. using $\boldsymbol{Z}$-gate transformation), and records the result, i.e. if he sees left or right glove in the mirror. The mirror image of the left glove is the right glove, and vice versa. If Alice tells Bob she sees two identical gloves in her two boxes, then Bob looks straight into his box and records what he sees. Surely enough, whatever result Bob records matches the content of the box which Alice puts aside, every time they repeat the experiment.

Similar protocol can be used to teleport a secret binary string χ of $N\ bits$. This protocol is known as *Vernam-Mauborgne one-time pad*. In this scenario, Bob generates a random string ψ of $N\ bits$ and secretly shares it with Alice, thus establishing a shared ancilla state. In order to teleport the string χ, Alice performs binary $\oplus$ (XOR) operation between string χ and her copy of the string ψ. Then, Alice reads bits of $\psi \oplus \chi$ string one by one. If Alice reads value 0 she tells Bob to read his bit as is. If Alice reads 1 she tells Bob to swap his bit into opposite. Even if Eve eavesdrops on Alice's communication to Bob, she would not be able to reconstruct secret string χ without having a copy of string ψ. The ancilla string ψ, just like the shared entangled state (2), serves as a *codebook*. The actual messages are transmitted via classical channel, but decrypted using shared codebook.

The thought experiments above prompt some legitimate questions as for the meaning of quantum teleportation and its potential usefulness, given the same results can be achieved in classical settings. Similar questions have been asked before [5]. For one thing, in any teleportation scheme, only $1\ bit$ of information about target state (1) is obtained by Bob per 1 preparation + measurement cycle. It requires the same $1\ bit$ of information (or $2\ bits$, if using publicized teleportation schemes), to be transmitted by Alice to Bob via classical channel. So, it seems nothing beyond what is transmitted via classical channel gets received by Bob.

References

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