No decoherence by entanglement

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There are misconceptions that entanglement (e.g. with environment) causes decoherence, and that decoherence causes classicality. Yet in an entanglement, barring classical communication, no action taken by one party has any effect on another party, a fact known as no-signaling theorem. The presented analysis reveals, it is the measurement, not entanglement, which turns quantum state into classical event sample, resulting in a loss of correlation terms of density matrix

One can read phrases like: *It is now widely accepted that environmental entanglement and the resulting decoherence processes play a crucial role in the quantum-to-classical transition and the emergence of "classicality" from quantum mechanics* [1]; *… the correlations of a quantum system with other quantum systems may cause one of its observables to behave in a classical manner* [2]. If one can cause an observable of a remote system "to behave in a classical manner" through entanglement, it would imply a *spooky action at a distance* [3, 4, 5], a long-discredited idea. Some of such misconceptions have been refuted [6, 7], some still persist.

Here I shall pinpoint the source of confusion about effects of entanglement. Consider a qubit $\boldsymbol{q}$ entanglement with an ancilla system $\boldsymbol{a}$ (e.g. environment), wherein qubit's eigenstates $|0\rangle$,$|1\rangle$ are correlated with ancilla's states $|u\rangle$,$|v\rangle$:

$$|\psi_+\rangle = \alpha|0,u\rangle + \beta|1,v\rangle \quad \text{, where } \alpha = \langle u,0|\psi_+\rangle \;\;;\;\; \beta = \langle v,1|\psi_+\rangle \;\;;\;\; \alpha^\dagger\alpha + \beta^\dagger\beta = 1 \tag{1}$$

The ancilla states $|u\rangle$,$|v\rangle$ are normalized but not necessarily orthogonal. The combined state (1) is pure, and its density matrix features interference terms, highlighted in purple:

$$\boldsymbol{\rho} = |\psi_+\rangle\langle\psi_+| = \alpha^\dagger\alpha|0,u\rangle\langle u,0| + \alpha^\dagger\beta|1,v\rangle\langle u,0| + \alpha\beta^\dagger|0,u\rangle\langle v,1| + \beta^\dagger\beta|1,v\rangle\langle v,1| \tag{2}$$

The standard approach [2, 8] to demonstrate how entanglement "causes decoherence" is to trace out ancilla from pure state density matrix (2) [9, 8]:

$$\boldsymbol{\rho_r} = Tr_{\boldsymbol{a}}(\boldsymbol{\rho}) = \alpha^\dagger\alpha|0\rangle\langle 0| + \alpha^\dagger\beta\langle u|v\rangle|1\rangle\langle 0| + \alpha\beta^\dagger\langle v|u\rangle|0\rangle\langle 1| + \beta^\dagger\beta|1\rangle\langle 1| \tag{3}$$

There are still interference terms, proportional to scalar product $\langle u|v\rangle$, in reduced density matrix (3). In a limit case, when $|u\rangle$,$|v\rangle$ are the same, i.e. when $\langle u|v\rangle = 1$, the reduced density matrix (3) is that of a standalone pure qubit state:

$$\boldsymbol{\rho_q} = \alpha^\dagger\alpha|0\rangle\langle 0| + \alpha^\dagger\beta|1\rangle\langle 0| + \alpha\beta^\dagger|0\rangle\langle 1| + \beta^\dagger\beta|1\rangle\langle 1| \tag{4}$$

In another limit, when $|u\rangle$,$|v\rangle$ are orthogonal, $\langle u|v\rangle = 0$, the interference terms in reduced density matrix (3) disappear, signifying full decoherence.

The expression for reduced density matrix (3) can be derived from measurement performed on state (1) with some arbitrary operator $\boldsymbol{X}$, acting on qubit only. The expectation value of the measurement is given by Born rule:

$$\begin{aligned}\langle\boldsymbol{X}\rangle = \langle\psi_+|\boldsymbol{X}|\psi_+\rangle &= \alpha^\dagger\alpha\langle 0|\boldsymbol{X}|0\rangle + \alpha^\dagger\beta\langle u|v\rangle\langle 0|\boldsymbol{X}|1\rangle + \alpha\beta^\dagger\langle v|u\rangle\langle 1|\boldsymbol{X}|0\rangle + \beta^\dagger\beta\langle 1|\boldsymbol{X}|1\rangle \\ &= Tr_{\boldsymbol{q}}(\boldsymbol{X}\boldsymbol{\rho_r})\end{aligned} \tag{5}$$

To some authors, the result (5) seems as a proof of decoherence by entanglement, because, in their view, the application of Born rule in (5) means the measurement performed on qubit only, with the result showing dependence on ancilla in the form of scalar product $\langle u|v\rangle$. Strangely though that a question is not asked, if such interpretation complies with no-signaling theorem [10]. Indeed, if the measurement only involves local qubit, the ancilla system in entangled state (1) can be chosen arbitrarily. It might not even physically exist, being just a gedanken. The expectation value of local measurement should not depend on gedanken, or remote system. Such dependence would mean the remote ancilla can somehow affect the measurement on local qubit. That runs contrary not just no-signaling theorem, but special relativity as well. Yet the result (5) depends on ancilla. So, what is the reason for paradox?

The culprit is hiding in plain view. Even though (5) is meant to be the measurement done with operator $\boldsymbol{X}$ on local qubit only, in fact, (5) contains two measurements, with two devices. One measurement is done with operator $\boldsymbol{X}$, and another measurement is done with identity operator $\boldsymbol{I}$:

$$\begin{aligned}\langle\boldsymbol{X}\rangle = \langle\psi_+|\boldsymbol{X}\cdot\boldsymbol{I}|\psi_+\rangle \\ &= \alpha^\dagger\alpha\langle 0|\boldsymbol{X}|0\rangle\langle u|\boldsymbol{I}|u\rangle + \alpha^\dagger\beta\langle 0|\boldsymbol{X}|1\rangle\langle u|\boldsymbol{I}|v\rangle + \alpha\beta^\dagger\langle 1|\boldsymbol{X}|0\rangle\langle v|\boldsymbol{I}|u\rangle \\ &+ \beta^\dagger\beta\langle 1|\boldsymbol{X}|1\rangle\langle v|\boldsymbol{I}|v\rangle = Tr_{\boldsymbol{q}}\big(\boldsymbol{X}\cdot Tr_{\boldsymbol{a}}(\boldsymbol{\rho})\big)\end{aligned} \tag{6}$$

The identity operator $\boldsymbol{I}$ is POVM operator in its own right. Born rule (6) for entangled state (1) implicitly includes measurement by identity operator (device $\boldsymbol{I}$). Generally, any expression in a form $\langle\psi|\boldsymbol{X}|\chi\rangle$ implies measurement, because its output is classical information. The application of Born rule (6) to (1) means the ancilla is part of a real physical system being measured, not a gedanken. It is the measurement by device $\boldsymbol{I}$ which turns pure state (2) into generally mixed state (3), not entanglement per se. As is evident from right side of (6), the subsequent measurement by device $\boldsymbol{X}$ is done not on pure state (2), but on output from the first measurement, the mixed state $\boldsymbol{\rho_r}$ (3). In the limit case of $\langle u|v\rangle = 0$, the measurement by device $\boldsymbol{I}$ completely turns pure state (2) into classical event sample. It eliminates any uncertainty about subsequent measurement by device $\boldsymbol{X}$, because orthogonal states $|u\rangle,|v\rangle$ each point [2] to the respective qubit eigenstate $|0\rangle,|1\rangle$.

The event sample from device $\boldsymbol{I}$ has to be available to device $\boldsymbol{X}$, i.e. classical information has to be shared between two devices. It explains why *spooky action at a distance*, or other claims of non-locality of quantum theory, is a fantasy, albeit still popular with some authors[1] [11, 7, 6]. The Born rule (6) imposes speed limit on two measurements in a form of requirement that the interval between them is timelike. If interval is spacelike, then classical information between two devices cannot be shared, and the measurement by device $\boldsymbol{X}$ has to be considered as single-device measurement on standalone qubit state (4), not two-device measurement on entangled state (2). This condition is equivalent to $\langle u|v\rangle = 1$ in (5,6), i.e. to inability of device $\boldsymbol{I}$ to distinguish between different ancilla states, or inability of device $\boldsymbol{I}$ to share classical information with device $\boldsymbol{X}$.

Consider a double-slit experiment with a beam of electrons. A charged particle, passing through the slit, would surely be detectable by sufficiently sensitive detector (device $\boldsymbol{I}$) placed next to the slit. The slit itself can act as a detector, if wired appropriately. In the absence of measurement at the slits one would see interference pattern at the screen (device $\boldsymbol{X}$) behind the slits. The interference signifies inability of device $\boldsymbol{X}$ to tell which slit the electron passed through. The interference is preserved even if the slits are wired to detect electrons, but the detected information

[1] The continuing controversy about non-locality of quantum theory has a lot to do with J.S. Bell making confusing statements on the subject, and an unfortunate EPR article [3]. The following catchy phrase [4] is definitely subject to misinterpretation: *grossly non-local structure... is characteristic… of any theory which reproduces exactly quantum mechanical predictions*

is not registered *anywhere* in classical form[2]. As soon as device $\boldsymbol{I}$ is turned on to register electrons passing through the slits, the interference pattern at device $\boldsymbol{X}$ will degrade, in accordance with reduced density matrix (3). The more accurate is the detection of electrons at the slits, the more orthogonal are states $|u\rangle,|v\rangle$, the smaller is the product $\langle u|v\rangle$, and the smaller are interference terms in (2,3,5,6).

The fact that measurement produces classical event sample, substantiates Bohr's postulate that the measuring device has to be classical: *[the] necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a principal distinction between classical and quantum-mechanical description of physical phenomena* [12].

Consequently, the measurement itself can be defined as extraction of classical information.

Thus, the classicality emerges as a result of measurement, in a form of classical event sample [13], not as a result of decoherence. As evident from (6), the decoherence itself is an aftermath of measurement. From this prospective, classicality and decoherence go hand in hand. No unitary process, described by, e.g. Schrödinger equation, can turn quantum state into classical, no matter how much interactions one plugs in[3], only measurement does.

The fact that entanglement itself does not produce decoherence or any other measurable effect has been attested in multitude of experiments on violation of Bell's inequalities, including those where measurements on two entangled entities are separated by spacelike interval [14, 15]. The strongest violation happens when no classical information is exchanged between two measuring devices, which corresponds to $\langle u|v\rangle = 1$ case in (5,6), i.e. when measurement by one device does not predetermine to any degree the measurement by another device[4].

The entanglement has to be considered a logical construct, indicating correlation, not causation, between measurements by different devices. The causation emerges when one measurement predetermines the results of another measurement. As I have shown, that happens when classical information is shared between measurement devices.

The claims that entanglement involves interaction can also be disproven[5]. The measurement result (6) does not depend on ancilla if measurement $\boldsymbol{X}$ is done in qubit eigenbasis, i.e. when $\langle 0|\boldsymbol{X}|1\rangle = \langle 1|\boldsymbol{X}|0\rangle = 0$. Such measurement would not detect interference. Why would interaction depend on how we choose to measure the qubit? The proponents of interaction explain this fact as follows: the ancilla (e.g. environment) enforces "effective superselection rules" [2] which select certain states to be "robust" against interaction. *What states are preferred will depend on the details of the interaction* [16]. Apparently, in case (1), ancilla chooses $|0\rangle,|1\rangle$ to be such robust states. However, by changing the measurement basis from $|0\rangle,|1\rangle$ to $|+\rangle,|-\rangle$, the experimenter can make

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[2] I refute the statement [16] that "*the pattern of detections at the screen cannot distinguish mere entanglement with some other systems from the actual use of those systems for detection at the slits*". It does distinguish, because, according to (6), without measurement of ancilla, there is no decoherence, no degradation of interference.

[3] The proof is trivial: the criterion for a pure quantum state $\boldsymbol{\rho}^2 = \boldsymbol{\rho}$ is invariant with respect to unitary transformations. Tracing out part of density matrix to illustrate decoherence is equivalent to performing measurement.

[4] One must acknowledge the numerous experiments on violation of Bell's inequalities are just variations of double-slit experiment, albeit done at much greater expense and effort.

[5] On high level the proof is trivial: the interaction by definition implies causation, which, as I have shown, only emerges as a result of measurement.

ancilla to "prefer" $|+\rangle, |-\rangle$. The measurement in $|+\rangle, |-\rangle$ basis, just like the measurement in $|0\rangle,|1\rangle$ basis would not detect interference:

$$|\psi_+\rangle = \alpha'|+\rangle|u'\rangle + \beta'|-\rangle|v'\rangle \quad ; \quad {\alpha'}^{\dagger}\alpha' + {\beta'}^{\dagger}\beta' = 1 \tag{7}$$

, where

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad ; \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{8}$$

$$|u'\rangle = \frac{\alpha|u\rangle + \beta|v\rangle}{\sqrt{2}\cdot\alpha'} \quad ; \quad |v'\rangle = \frac{\alpha|u\rangle - \beta|v\rangle}{\sqrt{2}\cdot\beta'} \tag{9}$$

$$\alpha' = \left(\frac{1 + \alpha^{\dagger}\beta\langle u|v\rangle + \alpha\beta^{\dagger}\langle v|u\rangle}{2}\right)^{1/2} \quad ; \quad \beta' = \left(\frac{1 - \alpha^{\dagger}\beta\langle u|v\rangle - \alpha\beta^{\dagger}\langle v|u\rangle}{2}\right)^{1/2} \tag{10}$$

There are infinitely many such measurement bases, which are "robust" against effects of entanglement. Since the bases which are "robust" depend on the will of experimenter, the assumption that they are selected by interaction between qubit and ancilla is false. The very notion of interaction as pertaining to physical reality independent of experimenter has to be abandoned.

One has to recognize that quantum mechanical expressions describe measurement setups, and predicted measurement outcomes, not abstractions like "quantum state", "entanglement", "superposition", "wave function", etc. E.g., the expression (1) describes configuration for two-device measurement. The expression $|\psi_+\rangle = \alpha|0\rangle + \beta|1\rangle$ describes single-device measurement, where $|\psi_+\rangle$, and $|\psi_-\rangle = \alpha|\beta/\alpha|\cdot|0\rangle - \beta|\alpha/\beta|\cdot|1\rangle$ are device eigenstates. As proved by Alain Aspect's famous experiment [14], the quantum state is not just unknown, it does not even exist prior to measurement.

The classical state, represented by an event sample, is encoded via a set of symbols from some alphabet [13]. Each symbol equates to a classical parameter value. E.g., the spatial coordinate of an object represents a [continuous] alphabet. The value of coordinate represents a symbol, produced as a result of measurement. No quantum object can have coordinate defined until the measurement is done, in coordinate eigenbasis. This dispels frequently heard claims that quantum particle can be at two different locations at the same time. It cannot, because the coordinate is not defined until the measurement. *It is not just unknown; it is not defined even as a parameter*. A presence of classical parameter in quantum mechanical expression, even implicit *hidden variable*, would indicate a carried-out measurement, just like the presence of scalar products $\langle\psi|\boldsymbol{X}|\chi\rangle$, $\langle u|v\rangle$.

The macroscopic objects are usually associated with classical behavior, i.e. with absence of interference. In view of things said, it can be explained as follows. Commonly accessible observables correspond to operators, which only act within small subset of object's degrees of freedom [17]. For simplicity, I consolidate those into Hilbert space formed by eigenstates $|0\rangle,|1\rangle$. Every vector from this Hilbert space is correlated with a vector from Hilbert space of the rest of object's degrees of freedom, e.g. $|0\rangle$ is correlated with $|u\rangle$, $|1\rangle$ is correlated with $|v\rangle$, as in (1). Due to the large number of degrees of freedom, macroscopically different states $|0\rangle,|1\rangle$ correspond to vectors $|u\rangle$, $|v\rangle$, which differ in many of the degrees of freedom [17], i.e. $|u\rangle$, $|v\rangle$ are orthogonal pointer states, $\langle u|v\rangle = 0$. This leads to vanishing interference terms in (5, 6).

To conclude, I have elucidated the origins of decoherence, classicality, and causation. I have argued that quantum mechanical expressions have to be watched for implicit measurements, which signify emerging classicality, such as the measurement on ancilla present in Born rule (6).

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